ST. THOMAS COLLEGE LIBRARY THIRUVANANTHAPURAM

(Pages : 3)

E – 4393

Reg. No. : .....

Name : .....

Second Semester B.Sc. Degree Examination, August 2018
First Degree Programme Under CBCSS
Complementary Course for Chemistry/Polymer Chemistry
MM – 1231.2 : MATHEMATICS – II
Integration, Differential Equations and Analytic Geometry
(2013 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the **first ten** questions are **compulsory**. They carry **1** mark **each**.

1. Evaluate $\int \sec x (\tan x + \cos x)\, dx$.
2. What is the surface area of the surface of revolution that is generated by revolving the portion of the curve $x = f(y)$ between $y = c$ and $y = d$ about the y-axis ?
3. Let $\int_1^4 f(x)\,dx = 2$ and $\int_1^4 g(x)\,dx = 10$. Find $\int_1^4 [3f(x) - g(x)]\,dx$.
4. Find the integrating factor of $\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$.
5. Solve $(D^2 - 6D + 9)y = 0$.
6. Solve $(x+1)\frac{dy}{dx} = 2e^{-y}$.
7. Write an equation for the parabola with vertex at (1, 1) and directrix at $y = -2$.
8. Find the equations to the asymptotes of the hyperbola $\frac{y^2}{9} - \frac{x^2}{25} = 1$.
9. Find the directrix of the parabola $x^2 = -9y$.
10. Write the parametric representation of the ellipse $\frac{x^2}{16} + \frac{y^2}{49} = 1$.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Find $F'(0)$ where $F(x) = \int_0^x \frac{\cos t}{t^2+3} dt$.

12. Find the area of the region enclosed by the parabola $y = 2x - x^2$ and the x-axis.

13. Find the arc length of the curve $x = \cos t + t \sin t$, $y = \sin t - t \cos t$, $0 \leq t \leq \pi$.

14. Write an equivalent integral of $\int_0^1 \int_{4x}^4 e^{-y^2} dy\, dx$ with the order of integration reversed.

15. Solve $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

16. Find the differential equation of the orthogonal trajectories of the system of parabolas $y = ax^2$, a being the parameter.

17. Find the particular integral of $(D^2 + 5D + 6)y = e^x$.

18. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$.

19. Find the vertices and ends of the minor axis of the ellipse $9(x-1)^2 + 16(y-3)^2 = 144$.

20. Using discriminant, identify the conic represented by the equation $x^2 - xy + y^2 - 2 = 0$.

21. Find a polar equation for an ellipse that has its focus at the pole whose eccentricity is $\frac{2}{3}$ and directrix $x = 1$.

22. Show that for an ellipse with eccentricity e, $\frac{r_1}{r_0} = \frac{1+e}{1-e}$.

SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. A ball is hit directly upward with an initial velocity of 49 m/s and is struck at a point that is 1 m above the ground. Assuming that the free-fall model applies, how high will the ball travel ?

24. Find the volume of the solid that results when the region enclosed by the curves $x = y^2$, $x = y + 2$ is revolved about the y-axis.

25. Use cylindrical coordinates to evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 dz\, dy\, dx$, $(a > 0)$.

26. Solve $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$.

27. Solve $(6xy + 2y^2 - 5)dx + (3x^2 + 4xy - 6)dy = 0$.

28. Find the orthogonal trajectories of the family of ellipses having center at the origin, a focus at the point (c, 0) and semimajor axis of length 2c.

29. Rotate the coordinate axes through an angle θ to remove the xy term from the equation $x^2 + 4xy - 2y^2 - 6 = 0$. Identify the new curve.

30. Find the equations of the tangents to the ellipse $9x^2 + 16y^2 = 144$ from the point (2, 3).

31. Find a polar equation of a hyperbola with a focus at the pole, eccentricity $\sqrt{2}$ and one of its vertices at (2, 0).

SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) Find the volume of the solid bounded by the surface $z = \sqrt{y}$ and the planes $x + y = 1$, $x = 0$ and $z = 0$.

    b) Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$ about the x-axis.

33. a) Solve $(x^2 - 3y^2)dx + 2xy\, dy = 0$.

    b) Find the particular integral of $(D^2 + 9)y = x \sin x$.

34. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(e^{2x} + \sin 2x + x^2)$.

35. a) Sketch the graph of $r = \frac{12}{4 + \cos\theta}$ in polar coordinates.

    b) Find the equations to the asymptotes of the hyperbola $8x^2 + 10xy - 3y^2 - 2x + 4y = 2$.